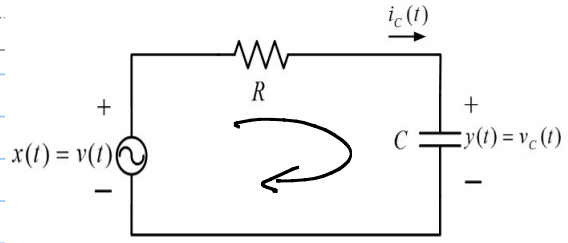
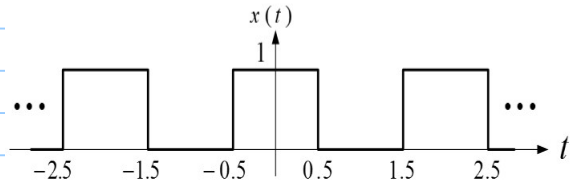
Note Title 4/2/2019





(1) Find the T.F $(H(\omega))$

$$-x(t) + R i_c(t) + y(t) = 0 \qquad ; \; i_c(t) = C \frac{dv_c(t)}{dt} = C \frac{dy(t)}{dt}$$

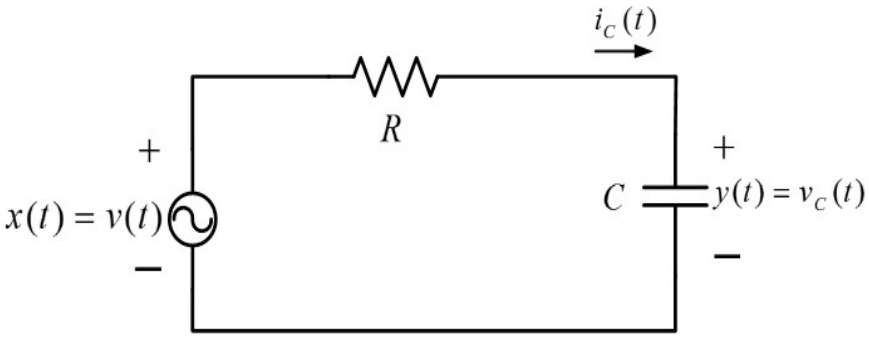
$$-x(t) + RC \frac{dy(t)}{dt} + y(t) = 0$$

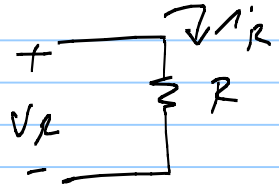
To find the T.F., assume all IC's are zeros

$$-X(\omega) + RC(j\omega) Y(\omega) + Y(\omega) = 0$$

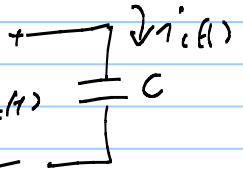
$$Y(\omega)[j\omega RC + 1] = X(\omega)$$

T.F. $\quad H(\omega) = \dfrac{Y(\omega)}{X(\omega)} = \dfrac{1}{1 + j\omega RC}$ OR $\quad H(\omega) = \dfrac{1/RC}{\frac{1}{RC} + j\omega}$





$$V_R(t) = R i_R(t)$$

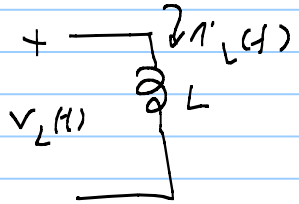


$$V_R(\omega) = R I(\omega)$$

$$i_c(t) = C \frac{dv_c(t)}{dt}$$

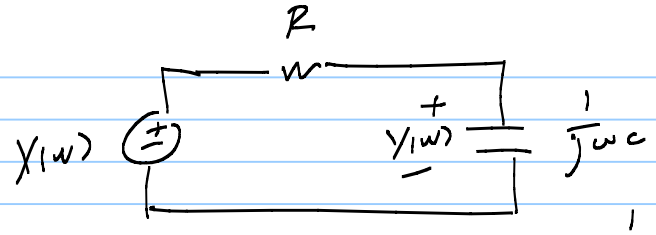
$$I_c(\omega) = j\omega C \, V_c(\omega)$$

$$V_c(\omega) = \frac{1}{j\omega C} I_c(\omega)$$



$$V_L(t) = L \frac{di_L(t)}{dt}$$

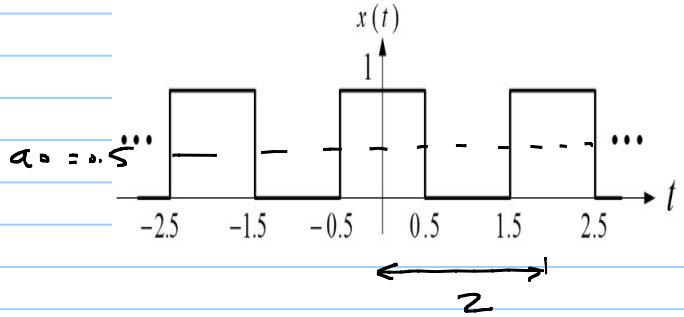
$$V_L(\omega) = j\omega L \, I_L(\omega)$$



$$Y(w) = X(w)\left(\frac{\frac{1}{jwC}}{R+\frac{1}{jwC}}\right)$$

$$\frac{Y(w)}{X(w)} = \frac{\frac{1}{jwC}}{R+\frac{1}{jwC}} = \frac{1}{1+jwRC}$$

(2)



$a_0 = 0.5$

$T = 2 \Rightarrow \omega_0 = \frac{2\pi}{T} = \pi$

$b_k = 0$ ⇐ Even function

$$a_k = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\cos(k\omega_0 t)\,dt = \frac{2}{2}\int_{-1}^{1} x(t)\cos(k\pi t)\,dt$$

$$= \int_{-0.5}^{0.5} (1)\cos(k\pi t)\,dt = \frac{1}{k\pi}\sin(k\pi t)\Big|_{-0.5}^{0.5}$$

$$= \frac{1}{k\pi}\left[\sin\left(\frac{k\pi}{2}\right) - \sin\left(\frac{-k\pi}{2}\right)\right]$$

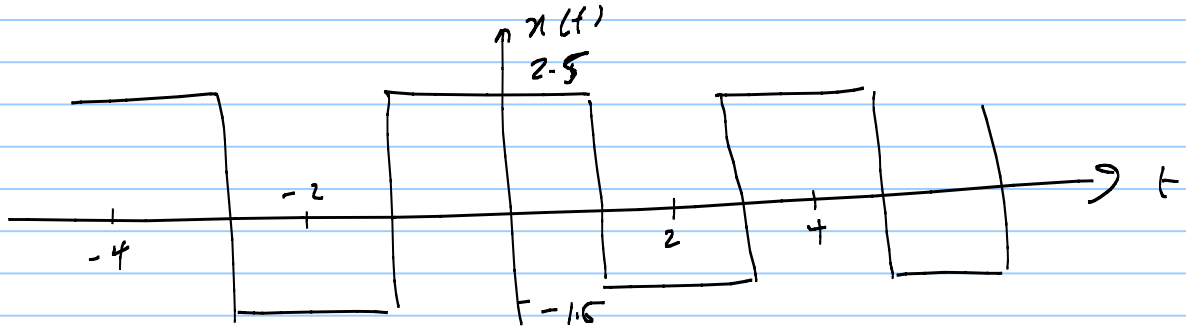
$$= \frac{1}{k\pi}\left[\sin\left(\frac{k\pi}{2}\right) + \sin\left(\frac{k\pi}{2}\right)\right] = \frac{2}{k\pi}\sin\left(\frac{k\pi}{2}\right)$$

$$x(t) = 0.5 + \sum_{k=1}^{\infty}\frac{2}{k\pi}\sin\left(\frac{k\pi}{2}\right)\cos(k\pi t), \quad -\infty < t < \infty$$

$$H(\omega) = \frac{1}{1+j\omega RC}$$

$$y(t) = 0.5\,H(0) + \sum_{k=1}^{\infty} \frac{2}{k\pi} \sin\left(\frac{k\pi}{2}\right) \left| H(k\pi) \right| \cos\left(k\pi t + \angle H(k\pi)\right)$$

$$= 0.5 + \sum_{k=1}^{\infty} \frac{2}{k\pi} \sin\left(\frac{k\pi}{2}\right) \frac{1}{\sqrt{1+(k\pi RC)^2}} \cos\left(k\pi t - \tan^{-1}(k\pi RC)\right)$$



$T = 4$; $a_0 = 0.5$, $b_k = 0$; $\omega_0 = \frac{\pi}{2}$

$$a_k = \frac{4}{T} \int_0^{T/2} x(t) \cos\left(k\frac{\pi}{2} t\right) dt$$

$$a_k = \int_0^1 2.5 \cos\left(\frac{k\pi}{2} t\right) dt - \int_1^2 1.5 \cos\left(\frac{k\pi}{2} t\right) dt$$

$$= \frac{2.5}{k\pi/2} \sin\left(\frac{k\pi}{2} t\right)\Big|_0^1 - \frac{1.5}{k\pi/2} \sin\left(\frac{k\pi}{2} t\right)\Big|_1^2$$

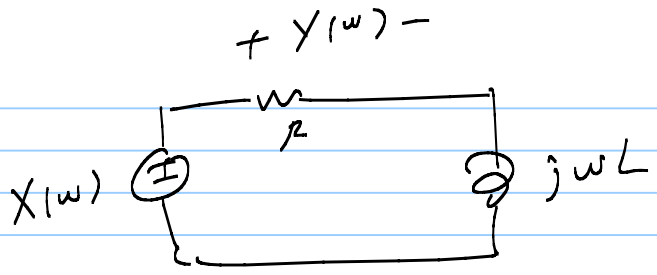
$$= \frac{5}{k\pi}\left[\sin\left(\frac{k\pi}{2}\right) - 0\right] - \frac{3}{k\pi}\left[\sin(k\pi) - \sin\left(\frac{k\pi}{2}\right)\right]$$

($\sin(k\pi) \to 0$)

$$= \frac{8}{k\pi} \sin\left(\frac{k\pi}{2}\right)$$

$$x(t) = 0.5 + \sum_{k=1}^{\infty} \frac{8}{k\pi} \sin\left(\frac{k\pi}{2}\right) \cos\left(k\frac{\pi}{2} t\right) \quad , -\infty < t < \infty$$

(1)



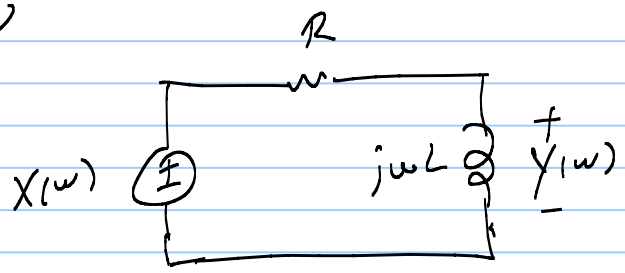
$$Y(w) = X(w)\left(\frac{R}{R+jwL}\right)$$

$$H(w) = \frac{R}{R+jwL} = \frac{R/L}{R/L+jw} = \frac{1000}{1000+jw}$$

$$y(t) = 0.5 + \sum_{k=1}^{\infty} \frac{8}{k\pi} \sin\left(\frac{k\pi}{2}\right)\left(\frac{R}{\sqrt{R^2+\left(\frac{k\pi}{2}L\right)^2}}\right) *$$

$$\cos\left(\frac{k\pi}{2}t + 0 - \tan^{-1}\left(\frac{\frac{k\pi}{2}L}{R}\right)\right)$$

(2)



$$H(w) = \frac{jwL}{R+jwL}$$

$$x(t) = 0.5 + \sum_{k=1}^{\infty} \frac{2}{k\pi} \sin\left(\frac{k\pi}{2}\right)\cos(k\pi t), \quad -\infty < t < \infty$$

$$y(t) = 0 + \sum_{k=1}^{\infty} \frac{2}{k\pi} \sin\left(\frac{k\pi}{2}\right)\left(\frac{\frac{k\pi}{2}L}{\sqrt{R^2+\left(\frac{k\pi}{2}L\right)^2}}\right)$$

$$\cos(\quad - \quad)$$

$$Y(w) = H(w)X(w)$$